PERGAMON
International Journal of Heat and Mass Transfer 42 (1999) 1895–1907
International Journal of HEAT and MASS TRANSFER

# Natural convective heat transfer from isothermal conic

W.M. Lewandowski*, S. Szymański, P. Kubski, E. Radziemska, H. Bieszk, T. Wilczewski

*Technical University of Gdańsk, Department of Apparatus and Chemical Machinery, ul.G.Narutowicza 11/12, 90-952 Gdańsk, Poland*

Received 29 December 1997; in final form 9 June 1998

**Abstract**

Theoretical considerations on convective heat transfer from isothermal upward conical surfaces have been presented. The physical model of this phenomenon consists of an isothermal cone of inclination angle ($\phi$) between the cone generating line ($X$) and the radius ($R$) of the cone base. The angle is a parameter of conical surface which varied from ($\phi = 0$ — circular horizontal plate) to ($\phi = \pi/2$—vertical cylinder). On the basis of Navier–Stokes equations, assuming the parabolic temperature profile in the boundary layer, the velocity profile tangent to the surface has been calculated. Introduction of the mean velocity value in the boundary layer into the balance of energy and mass equations and comparison with the Newton equation leads to the dependence describing the boundary layer thickness. Next the relation of Nusselt and Rayleigh numbers, including a function expressing the influence of the inclination angle ($\phi$) on the heat transfer process, has been derived. The obtained solution describes the natural convective heat transfer process for three characteristic cases of the conical surface.

For the boundary cases $\phi = \pi/2$ (vertical cylinder) and $\phi = 0$ (circular upward facing horizontal plate), the solution describing convective heat transfer intensity is

$Nu_{X=H} = 0.668 \cdot Ra_{X=H}^{1/4}$ for $\phi = \pi/2$ and $Nu_{X=R} = 0.932 \cdot Ra_{X=R}^{1/5}$ for $\phi = 0$.

For the case ($0 < \phi < \pi/2$) (cones), the solution has the form

$Nu_X = 1.680 \cdot \Phi^{1/4} \cdot Ra_X^{1/4}$

where ($\Phi$) is a function of the inclination angle ($\phi$) of the generating line of the conical surface to the base of radius ($R$).


**Nomenclature**

$a = \lambda/(c_p\rho)$ thermal diffusivity
$A$ cross-section area in boundary layer (3)
$A_k$ control surface on cone (4)
$C$ coefficient in Nusselt–Rayleigh relation
$C_0$, $C_1$ constants in equations (18) and (19)
$F = d\delta/dx = d\delta^*/-dR^*$ coefficient of boundary layer shape (27)
$h$ enthalpy
$h1$ thickness of copper–epoxy resin–copper laminate in the second set-up
$K$ constant (12)
$m$ mass flux
$Nu = \alpha D/\lambda = \alpha R/\lambda$ Nusselt number
$Q$ heat flux
$r$, $R$ radius, radius of the base cone
$Ra = g\beta\Delta TR^3/(\nu a)$ Rayleigh number
$T$ temperature
$u$ function (16)
$W$ velocity
$x$, $y$ tangential and normal co-ordinates to the surface
$X$ characteristic linear dimension, element of the cone, height of cylinder.

*Greek symbols*
$\alpha$ heat transfer coefficient
$\delta$ thickness of boundary layer

* Corresponding author. Fax: 0048 58 347 2410; e-mail: wlew@altis.chem.pg.gda.pl

0017–9310/98/$ - see front matter © 1998 Elsevier Science Ltd. All rights reserved
PII: S0017–9310(98)00283–X

$\lambda$ conductivity of the fluid
$\lambda 1$ conductivity of epoxy–copper laminate
$\nu$ kinematic viscosity
$\phi$ inclination angle of the generating line of the conical surface
$\Phi$ the function defined by equation (26).

## 1. Introduction

Results of experimental and theoretical investigations of heat transfer from conical surfaces are not often published as from spherical, flat or cylindrical—horizontal, vertical and inclined surfaces, because these surfaces are not widely used in technical applications. They are represented in civil engineering (towers), chemical industry (tanks, bottoms and covers of tanks), electronics (transistors, resistors, diodes, lamps). It is still a problem to calculate the intensity of heat transfer, especially free convective heat transfer from conical surfaces.

Works, e.g. [1–6], concerning cones are not complete.

The aim of the work is a presentation of an experimentally verified analytical solution describing the natural convective heat transfer from conical surfaces in the full range of the inclination angle ($\phi$) of the generating line of the considered surface. The angle varies from $\phi = 0$ (circular upward facing horizontal plate to $\phi = \pi/2$ (vertical cylinder).

## 2. The physical model

The physical model of the natural convection heat transfer from conic shown in Fig. 1 is proposed as a consequence of visualisation of this phenomenon, presented as an example in Fig. 9. The solution of the Fourier–Kirchhoff equation with typical simplifying assumptions in the form of the dependence of the temperature profile in the boundary layer (1) after introduction into the Navier–Stokes equations for an inclined boundary layer has led to the relation of the local and next to the mean value of the fluid velocity in a parallel direction to an isothermal surface (2). The entire derivation is given in the paper [7].

$$\Theta = \frac{(T-T_\infty)}{(T_w - T_\infty)} = \left(1-\frac{y}{\delta}\right)^2 \tag{1}$$

$$\bar{W}_x = \frac{1}{\delta}\int_0^\delta W_x\,dy = \frac{g\beta\Delta T\delta^2}{\nu}\left(\frac{\partial\delta}{\partial x}\frac{\cos\phi}{72}+\frac{\sin\phi}{40}\right). \tag{2}$$

The control surfaces ($A$) for the mass flux of the fluid and ($dA_k$) for the heat flux in boundary layer above the cone are as follows:

$$A = 2\cdot\pi\cdot r\cdot\delta = 2\cdot\pi\cdot\delta\cdot(R-x\cos\phi) = 2\cdot\pi\cdot\delta\cdot\cos\phi\cdot(X-x) \tag{3}$$

$$dA_k = 2\cdot\pi\cdot r\cdot dx = 2\cdot\pi\cdot(R-x\cos\phi)\cdot dx = 2\cdot\pi\cdot\cos\phi\cdot(X-x)\cdot dx. \tag{4}$$

From the continuity equation of the mass flux inside the boundary layer the value of the heat flux transported in the fluid arise:

$$dQ = \Delta h\,dm = c_p(T-T_\infty)_{mean}\,d(\rho\bar{W}_x A). \tag{5}$$

The mean value of the temperature difference within the boundary layer is:

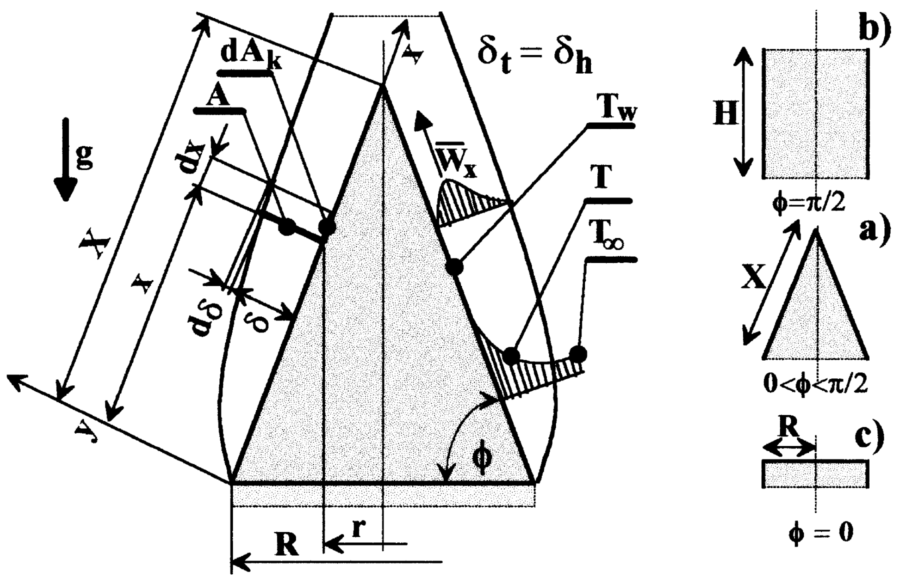


Fig. 1. The physical model of convective heat transfer from conical surface.

$$(T_x - T_\infty)_{\text{mean}} = \frac{1}{\delta}\int_0^\delta \Delta T\left(1-\frac{y}{\delta}\right)^2 \mathrm{d}y = \frac{\Delta T}{3}. \tag{6}$$

Introduction of equations (6), (2) and (3) to equation (5) gives:

$$\mathrm{d}Q = \frac{2\pi\rho g\beta c_\text{p}\cos\phi(\Delta T)^2}{3v}\mathrm{d}\left[\delta^3(X-x)\left(\frac{\partial\delta}{\partial x}\frac{\cos\phi}{72}+\frac{\sin\phi}{40}\right)\right]. \tag{7}$$

According to Newton's law the heat stream transferred from the control surface of the cone ($\mathrm{d}A_k$) is proportional to both, the coefficient of heat transfer ($\alpha$) and the temperature difference ($\Delta T$):

$$\mathrm{d}Q = \alpha\cdot\Delta T\cdot\mathrm{d}A_k = -\lambda\left(\frac{\partial\theta}{\partial y}\right)_{y=0}\Delta T\cdot 2\pi\cos\phi(X-x)\,\mathrm{d}x. \tag{8}$$

The dimensionless temperature gradient on the heating surface can be derived from the temperature profile (1):

$$\left(\frac{\partial\theta}{\partial y}\right)_{y=0} = -\frac{2}{\delta}. \tag{9}$$

Comparison of equations (7) and (8), after introduction of equation (9) leads to:

$$\frac{\rho g\beta c_\text{p}\Delta T\delta}{6v}\mathrm{d}\left[\delta^3(X-x)\left(\frac{\partial\delta}{\partial x}\frac{\cos\phi}{72}+\frac{\sin\phi}{40}\right)\right] = (X-x)\,\mathrm{d}x. \tag{10}$$

To obtain the equation (10) in dimensionless form, the following parameters have been introduced

$$\delta^* = \frac{\delta}{X},\quad X^* = \frac{X-x}{X} = 1-\frac{x}{X},$$

$$\Phi = \frac{\partial\delta^*}{-\partial X^*}\frac{\cos\phi}{72}+\frac{\sin\phi}{40} \cong \text{const},$$

$$Ra = \frac{g\beta(T_\text{w}-T_\infty)X^3}{va}. \tag{11}$$

The result is written in the form of equation (12)

$$\frac{\delta^*}{X^*}\frac{\mathrm{d}(\delta^{*3}X^*)}{\mathrm{d}X^*} = -\frac{6}{\Phi\,Ra} = -K. \tag{12}$$

Successive steps of the solution of equation (12) are as follows:

$$\frac{\delta^*}{X^*}\left(\frac{3\delta^{*2}\,\mathrm{d}\delta^*}{\mathrm{d}X^*}+\delta^{*3}\right) = -K \tag{13}$$

$$\frac{3\delta^{*3}}{X^*}\frac{\mathrm{d}\delta^*}{\mathrm{d}X^*}+\frac{\delta^{*4}}{X^*} = -K \tag{14}$$

$$\frac{3}{4}\frac{\mathrm{d}u}{\mathrm{d}X^*}+\frac{u}{X^*} = -1 \tag{15}$$

where

$$u = \frac{\delta^{*4}}{K}. \tag{16}$$

The solution of the nonhomogeneous differential equation (15) can be obtained in the first step, as a solution of the homogeneous equation:

$$\frac{3}{4}\frac{\mathrm{d}u}{\mathrm{d}X^*}+\frac{u}{X^*} = 0 \tag{17}$$

in the form:

$$u = \frac{C_0}{X^{*4/3}} \tag{18}$$

and next, after introducing the constant of integration ($C_0 = C_0(X^*)$), differentiation of equation (18) and introducing the result into equation (17) one obtains:

$$u = -\frac{4}{7}X^*+\frac{C_1}{X^{*4/3}}. \tag{19}$$

For the boundary condition $X^* = 1$, $u = 0(\delta = 0)$ and ($C_1 = 4/7$) the solution—dimensionless boundary layer thickness—can be expressed as:

$$u = \frac{4}{7}\frac{(1-X^{*7/3})}{X^{*4/3}} \tag{20}$$

and next, regarding introductions (12) and (16):

$$\delta^* = \left(\frac{24}{7}\right)^{1/4}\frac{(1-X^{*7/3})^{1/4}}{\Phi^{1/4}\cdot Ra^{1/4}\cdot X^{*1/3}}. \tag{21}$$

Based on the dimensionless boundary layer thickness connected with the coefficient of heat transfer by ($\alpha = 2\lambda/\delta$) local and next the mean relation of Nusselt and Rayleigh numbers can be evaluated:

$$Nu_X = \left(\frac{14}{3}\right)^{1/4}\frac{X^{*1/3}}{(1-X^{*7/3})^{1/4}}\Phi^{1/4}\,Ra_X^{1/4}. \tag{22}$$

The projection of the cone side surface on the base allows its transformation to circular surface. This facilitates the averaging of local values to the global ones.

Non-dimensional linear dimension built on the base of the cone generating line is equivalent to the dimension built on the base of the cone radius ($X^* = R^*$). This results from relation ($X^* = \cos\phi\cdot(X-x)/X = (R-x\cdot\cos\phi)/R = R^*$). In a considered circular surface its centre point is represented by ($R^* = 0$) the diameter is ($R^* = 1$).

As a result of such a transformation the averaging of equation (22) gives:

$$N\bar{u}_X = \frac{1}{S}\int_{(S)} Nu_X 2\pi R^* \, \mathrm{d}R^*$$

$$= \left(\frac{224}{3}\right)^{1/4} \Phi^{1/4} \, Ra_X^{1/4} \int_0^1 \frac{R^{*4/3}}{(1-R^{*7/3})^{1/4}} \mathrm{d}R^*. \quad (23)$$

The integral in equation (23) has a solution in the form of elementary functions:

$$\int_0^1 \frac{R^{*4/3}}{(1-R^{*7/3})^{1/4}} \mathrm{d}R^* = \left[\frac{4}{7}(1-R^{*7/3})^{3/4}\right]_0^1 = \frac{4}{7} \quad (24)$$

and therefore:

$$N\bar{u}_X = 1.680 Ra_X^{1/4} \, \Phi^{1/4}. \quad (25)$$

It has been assumed that the function in equation (11)

$$\Phi = F\frac{\cos\phi}{72} + \frac{\sin\phi}{40} \cong \text{const} \quad (26)$$

has a constant value for a definite inclination angle ($\phi$). This assumption is valid when the coefficient ($F$), defining the velocity of the boundary layer growth is averaging along the direction of the boundary layer growth.

$$\bar{F} = \frac{1}{S}\int_{(S)} \frac{\mathrm{d}\delta^*}{-\mathrm{d}R^*} 2\pi R^* \, \mathrm{d}R^*$$

$$= -2\int_0^1 R^* \, \mathrm{d}\delta^* = -2\left([\delta^* R^*]_0^1 - \int_0^1 \delta^* \, \mathrm{d}R^*\right)$$

$$= -\frac{2}{\Phi^{1/4} \, Ra_x^{1/4}} \left(\frac{24}{7}\right)^{1/4} \left([(1-R^{*7/3})^{1/4} R^{*2/3}]_0^1 - \int_0^1 \frac{(1-R^{*7/3})^{1/4}}{R^{*1/3}} \mathrm{d}R^*\right)$$

$$= \frac{2.7215}{\Phi^{1/4} \, Ra_X^{1/4}} \int_0^1 \frac{(1-R^{*7/3})^{1/4}}{R^{*1/3}} \mathrm{d}R^*. \quad (27)$$

The integral in equation (27), according to the Czybyszew rule, has no solution in the form of elementary functions, so the subintegral term is given as the Taylor series

$$(1-R^{*7/3})^{1/4} \cong 1 - \tfrac{1}{4}R^{*7/3} - \tfrac{3}{32}R^{*14/3} - \tfrac{7}{128}R^{*21/3} - \tfrac{77}{2048}R^{*28/3} \ldots \quad (28)$$

Introducing equation (28) into integral one obtains

$$\int_0^1 \left(\frac{1}{R^{*1/3}} - \frac{1}{4}R^{*2} - \frac{3}{32}R^{*13/3} - \frac{7}{128}R^{*20/3} - \frac{77}{2048}R^{*9}\ldots\right)\mathrm{d}R^* = \left[\frac{3}{2}(1-R^{*2/3}) - \frac{1}{12}(1-R^{*3}) - \frac{9}{512}(1-R^{*16/3}) - \frac{21}{2944}(1-R^{*23/3}) - \frac{77}{20\,480}(1-R^{*10})\ldots\right]_0^1 = 1.3882. \quad (29)$$

Introduction of the integral value (29) into equation (27) gives:

$$F\left(F\frac{\cos\phi}{72} + \frac{\sin\phi}{40}\right)^{1/4} = \frac{3.778}{Ra_X^{1/4}}. \quad (30)$$

For the two boundary cases ($\phi = 0°$)—circular upward facing horizontal plate and ($\phi = \pi/2$)—vertical cylinder, the transformation of equations (25), (26) and (30) leads to:

$$Nu_{X=R} = 0.932 \cdot Ra_{X=R}^{1/5} \quad \text{for horizontal circular plate (Fig. 1c)} \quad (31)$$

$$Nu_{X=H} = 0.668 \cdot Ra_{X=H}^{1/4} \quad \text{for vertical cylinder (Fig. 1b).} \quad (32)$$

The other cases of conical surfaces of inclination angle ($0 < \phi < \pi/2$) (Fig. 1a) need a solution of a set of two equations (26) and (30) with the two unknown ($F$) and ($\Phi$). The results of such calculations performed for given values of the Rayleigh number ($Ra_X = 10^3$, $10^4$, $10^5$, $10^6$, $10^7$ and $10^8$) are presented in Fig. 2.

The analytical solution presented above was obtained with an assumption that both thermal and hydraulic boundary layer thicknesses are of similar values ($\delta_T \approx \delta_h$) and that dissipative terms of the Navier–Stokes equations were neglected. This means that the solution is valid for Prandtl number ($Pr \approx 1$) and for laminator range (Rayleigh number, $Ra < 10^8$).

## 3. Experimental apparatus

The experimental studies were performed in two set-ups using three fluids: dehydrated glycerine, distilled water and air. The cross-section of the first set-up having a tank of 0.5 m in diameter and 0.4 m in height is presented in Fig. 3. In this set-up designed and constructed to carry out visualisation experiments of convective flow structures of the fluid, five cones [$\phi = 0$ (horizontal plate), $\pi/6$, $\pi/4$, $\pi/3$ and $3\cdot\pi/8$] were tested in glycerine. The diameter of the base of these cones were constant ($D = 0.06$ m).

The apparatus was also used to study the natural convective heat transfer from an isothermal hemisphere and complex surfaces and more details on this set-up and about the procedure of estimation of heat fluxes ($Q_{out}$), ($Q_{inl}$) and ($Q_{los}$) is published in the papers [8, 18].

The second apparatus shown in Fig. 4, was designed to perform quantitative heat transfer experiments of high accuracy for determination of the coefficient ($\alpha$) $\pm 6.3\%$ (water) and $\pm 5.3\%$ (air). Determination of the Nusselt number was accomplished with an error of $\pm 6.6\%$

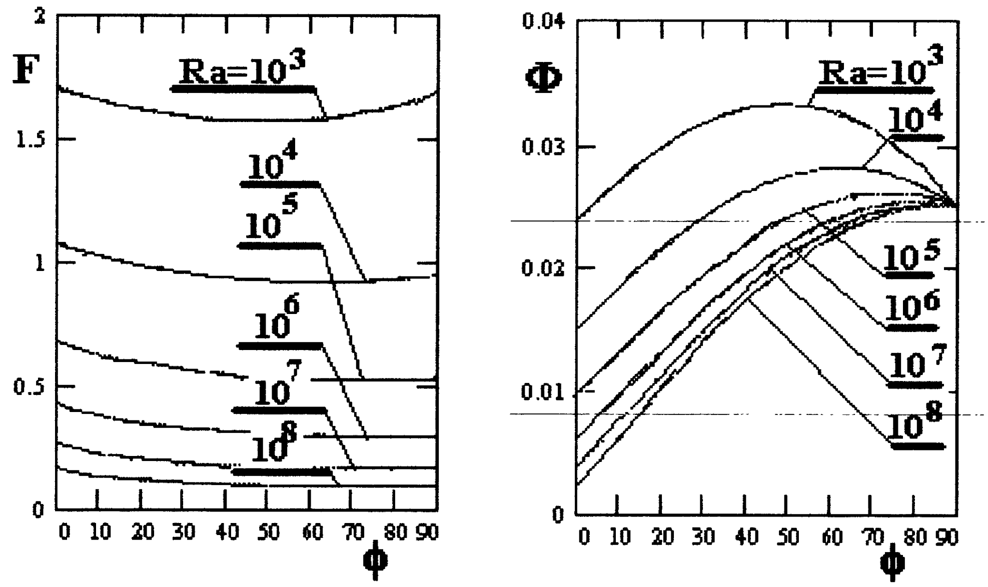


Fig. 2. Dependence of angle ($\phi$) and the Rayleigh number ($Ra_R$) on the boundary layer shape ($F$) and the function ($\Phi$) in equation (25).

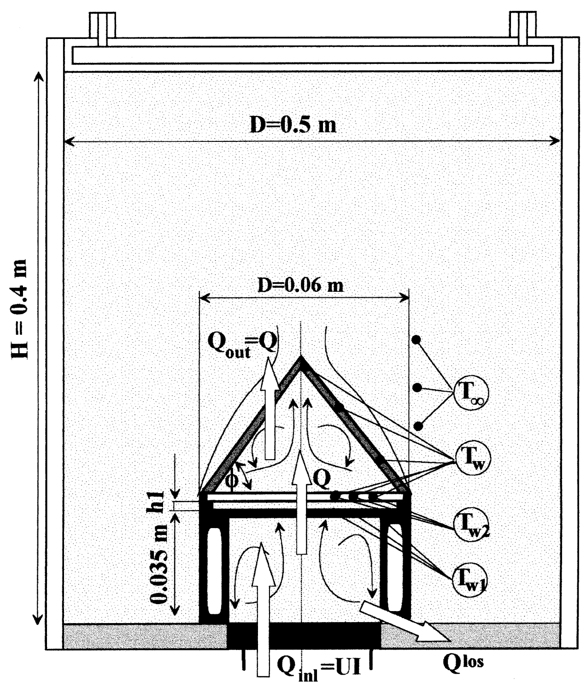


Fig. 3. Cross-section of the set-up for measuring convective heat transfer in glycerine.

(water) and $\pm 5.6\%$ (air). For the Rayleigh number the error values were $\pm 4.3\%$ (water) and $\pm 2.1\%$ (air).

The main part of it is a glass cuboidal tank of dimensions: $0.5 \times 0.5$ m (the base) and 0.8 m of height. The copper cones of the diameter ($D = 0.1$ m) and of angles by the base ($\phi = \pi/3$ and $5 \cdot \pi/12$) were placed in the tank. They were heated by means of an electric heater. Thermocouples copper–constantan were used to measure the temperature. The cone surface temperature ($T_w$) was measured by three thermocouples placed inside so that the welded measuring connections were exactly at the cone side surface. Also three thermocouples measured the fluid temperature in the undisturbed region ($T_\infty$). The temperature of both the cone surface and the undisturbed region were calculated as the average value of all the particular thermocouples.

The measuring plate of a special layer construction consisted of two circular copper plates and of epoxy-resin circular plate of a known thickness ($h1$) and the heat conduction coefficient ($\lambda$) between them was used to measure the back heat losses flux ($Q_{los}$). The thermal coefficient of conductivity for the laminate (copper–epoxy-resin–copper) was experimentally determined at a specially constructed stand. The relation describing the thermal coefficient of conductivity [$\lambda 1 = f(T)$ was obtained with an error of ($\lambda 1 = 0.2912 + 0.021 \cdot (T_1 + T_2)/2) \pm 2.5\%$]. Based on temperatures ($T_1$) and ($T_2$) of two copper plates measured by thermocouples placed inside the measuring plate the heat flux was calculated as:

$$Q = U \cdot I - Q_{los} \tag{33}$$

where

$$Q_{los} = \frac{\pi \cdot D^2}{4} \frac{\lambda 1}{h1} (T_1 - T_2). \tag{34}$$

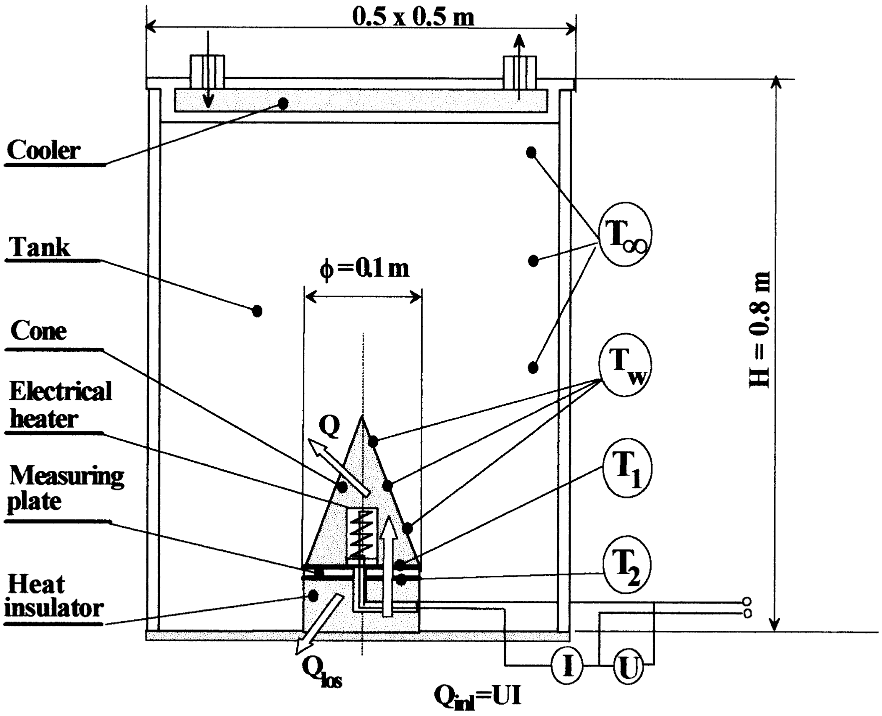


Fig. 4. Cross-section of the set-up for measuring convective heat transfer in water and air.

Investigations in the second set-up were carried out in distilled water and in the air. In the last case the radiative heat losses flux from the conical surface was also regarded. In calculations the emissivity coefficient for Polish copper was assumed.

## 4. Experimental results

Experimental results for glycerine recorded at the first stand are presented in Fig. 5. Analytical solutions of equations (25), (26) and (30) obtained for definite angles ($\phi$) are marked by a solid line.

Experimental results obtained for a circular upward facing horizontal plate were first approximated using the least square method without giving the value of the exponent ($n$). For comparison with literature data, the approximative procedure was repeated, but this time for a given value of the exponent ($n = 1/5$). Using the same method and the value of the exponent ($n = 1/4$) it is possible to compare all cases of ($\phi$) in one diagram (Fig. 8). For the ($\phi = 0$) the following relations were obtained:

$$Nu_{X=R} = 0.934 \cdot Ra_R^{0.204} (\delta^2 = 0.995),$$

$$Nu_{X=R} = 0.973 \cdot Ra_R^{1/5} \quad \text{and}$$

$$Nu_{X=R} = 0.549 \cdot Ra_R^{1/4}. \tag{35}$$

The next results obtained have the form:

$$Nu_X = 0.960 \cdot Ra_X^{0.214} (\delta^2 = 0.987) \quad \text{and}$$
$$Nu_X = 0.579 \cdot Ra_R^{1/4} \quad \text{for } \phi = \pi/6 \tag{36}$$

$$Nu_X = 1.023 \cdot Ra_X^{0.217} (\delta^2 = 0.997) \quad \text{and}$$
$$Nu_x = 0.641 \cdot Ra_R^{1/4} \quad \text{for } \phi = \pi/4 \tag{37}$$

$$Nu_X = 0.990 \cdot Ra_X^{0.227} (\delta^2 = 0.998) \quad \text{and}$$
$$Nu_X = 0.705 \cdot Ra_R^{1/4} \quad \text{for } \phi = \pi/3 \tag{38}$$

$$Nu_X = 0.929 \cdot Ra_X^{0.229} (\delta^2 = 0.989) \quad \text{and}$$
$$Nu_X = 0.670 \cdot Ra_R^{1/4} \quad \text{for } \phi = 3 \cdot \pi/8. \tag{39}$$

The experiments for glycerine as a tested fluid were performed within the range of Rayleigh numbers; $3.5 \cdot 10^4 < Ra_X < 3 \cdot 10^7$ and of Prandtl numbers; $256 < Pr < 1235$.

Experimental results recorded at the second stand are approximated by the following relations:

$$Nu_X = 0.975 \cdot Ra_X^{0.228} (\delta^2 = 0.996) \quad \text{and}$$
$$Nu_X = 0.674 \cdot Ra_R^{1/4} \quad \text{for } \phi = \pi/3 \quad \text{(water)} \tag{40}$$

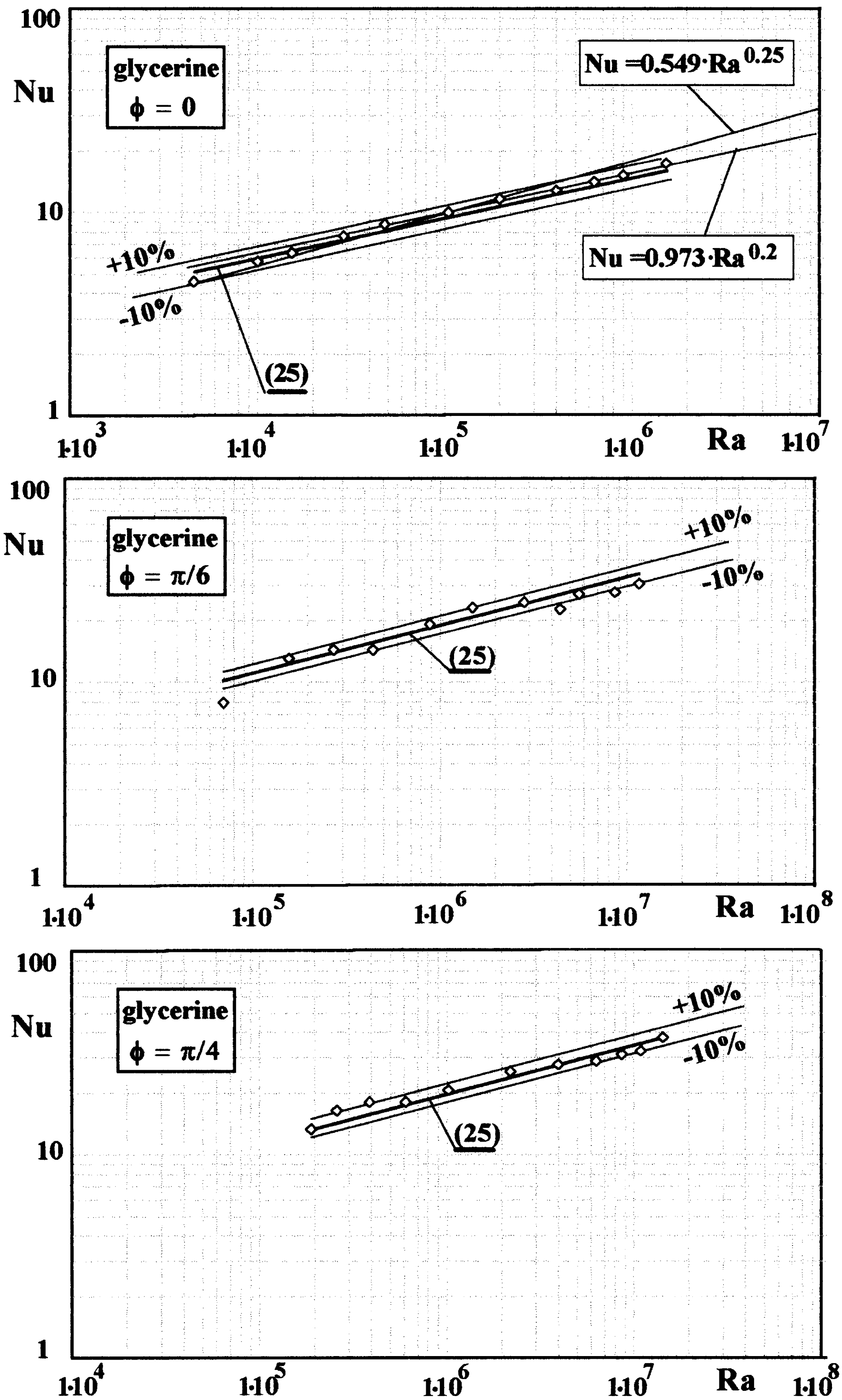


Fig. 5. Experimental results of convective heat transfer from conical surfaces obtained in the first set-up in glycerine.

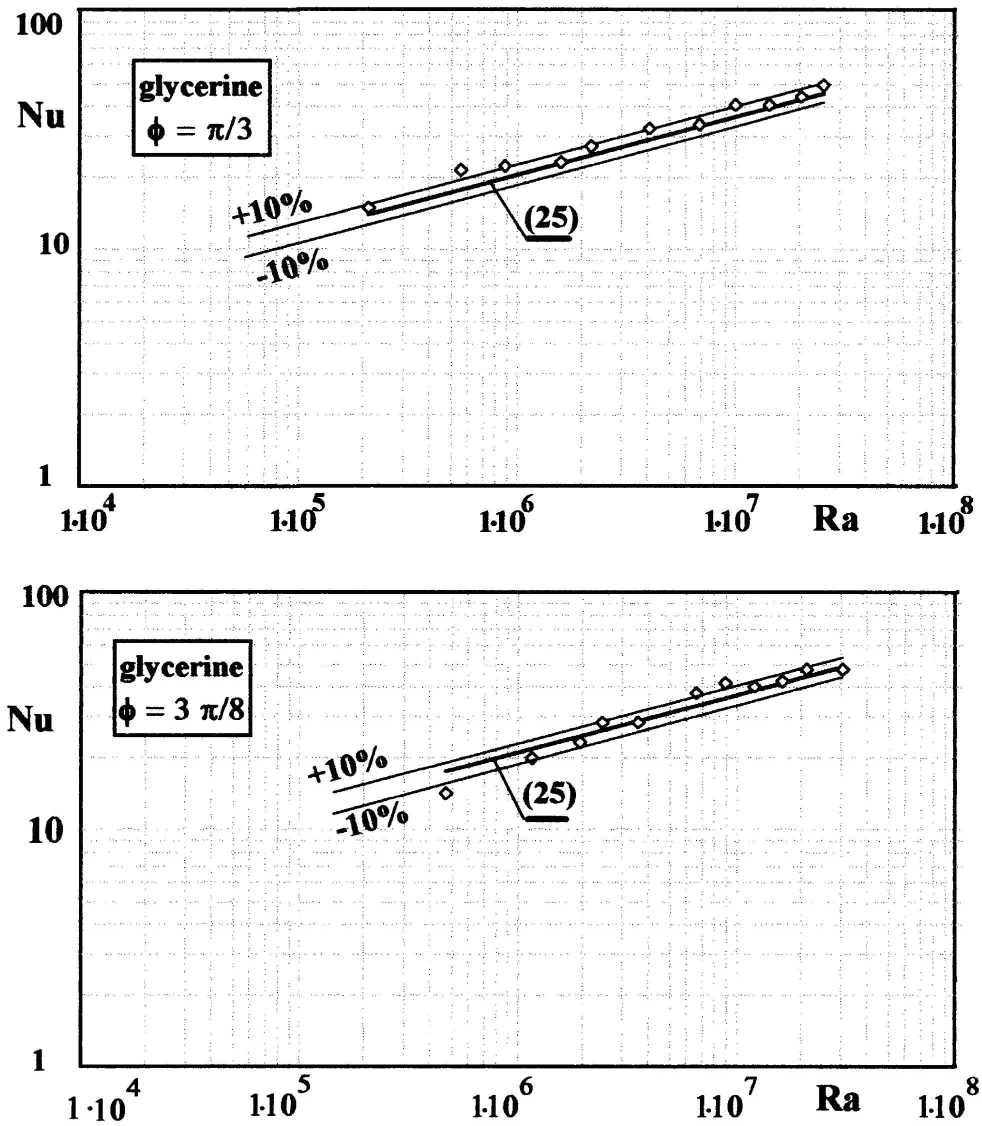


Fig. 5—*continued.*

$$Nu_X = 1.008 \cdot Ra_X^{0.217} (\delta^2 = 0.997) \quad \text{and}$$

$$Nu_X = 0.632 \cdot Ra_R^{1/4} \quad \text{for } \phi = \pi/3 \quad \text{(air)} \qquad (41)$$

$$Nu_X = 0.979 \cdot Ra_X^{0.236} (\delta^2 = 0.990) \quad \text{and}$$

$$Nu_X = 0.748 \cdot Ra_R^{1/4} \quad \text{for } \phi = 5 \cdot \pi/12 \quad \text{(water)} \qquad (42)$$

$$Nu_X = 0.987 \cdot Ra_X^{0.222} (\delta^2 = 0.995) \quad \text{and}$$

$$Nu_X = 0.635 \cdot Ra_R^{1/4} \quad \text{for } \phi = 5 \cdot \pi/12 \quad \text{(air)} \qquad (43)$$

and are illustrated in Fig. 6 for water and in Fig. 7 for air.

The appropriate range obtained for water was of: $3.8 \cdot 10^6 < Ra_X < 9.3 \cdot 10^8$ and $5.89 < Pr < 6.95$. For the air the range is as follows: $5.1 \cdot 10^5 < Ra_X < 1.4 \cdot 10^7$ and $0.697 < Pr < 0.702$.

All experimental results are expressed in a form of constant $C$ in Nusselt–Rayleigh relations

$$C = \frac{Nu_X}{Ra_X^{1/4}} \qquad (44)$$

compared with our analytical solution equations (25), (26) and (30) (solid line) and the solution of Stewart for a vertical isothermal cone in conditions of high Prandtl numbers ($Pr \to \infty$) or of Hering for low Prandtl numbers equation (45) (dotted line for $Pr = 100$) given in [9] are collected in Fig. 8.

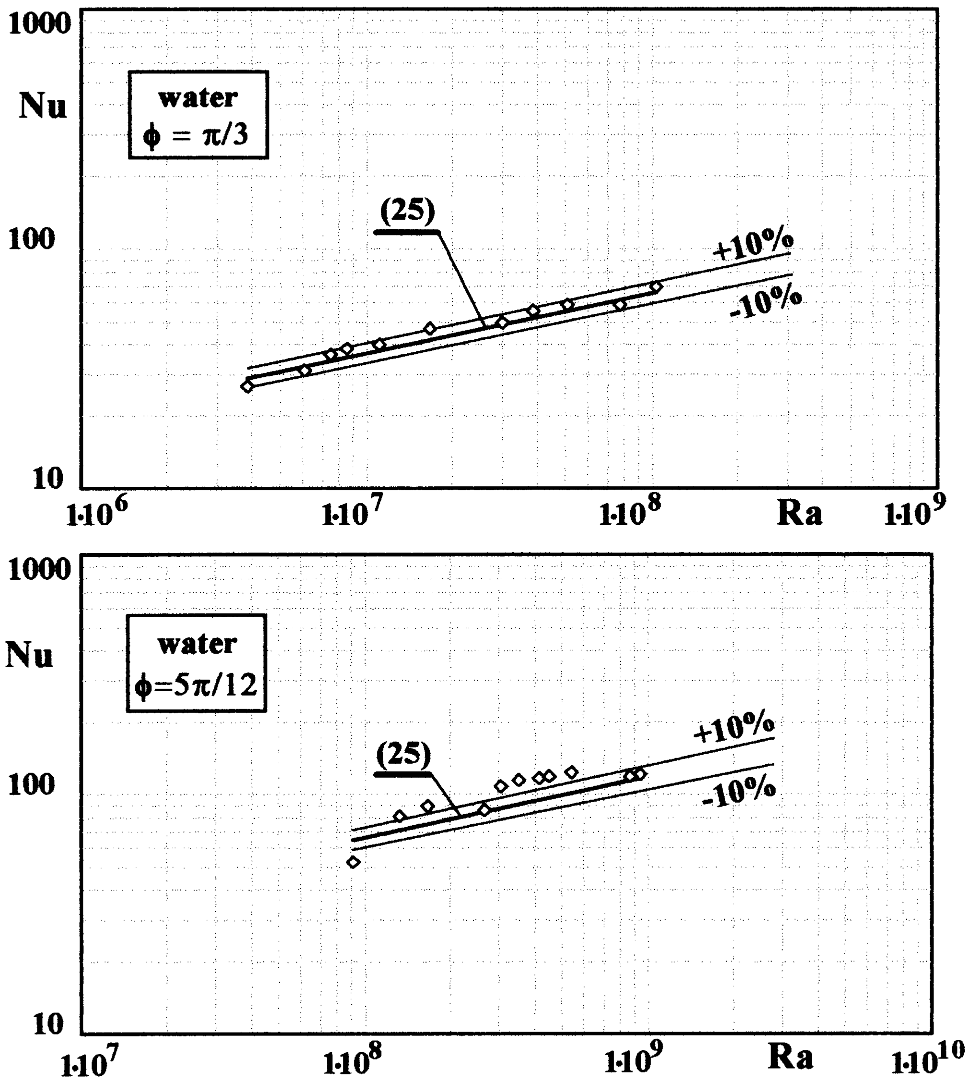


Fig. 6. Experimental results of convective heat transfer from conical surfaces obtained in the second set-up in water.

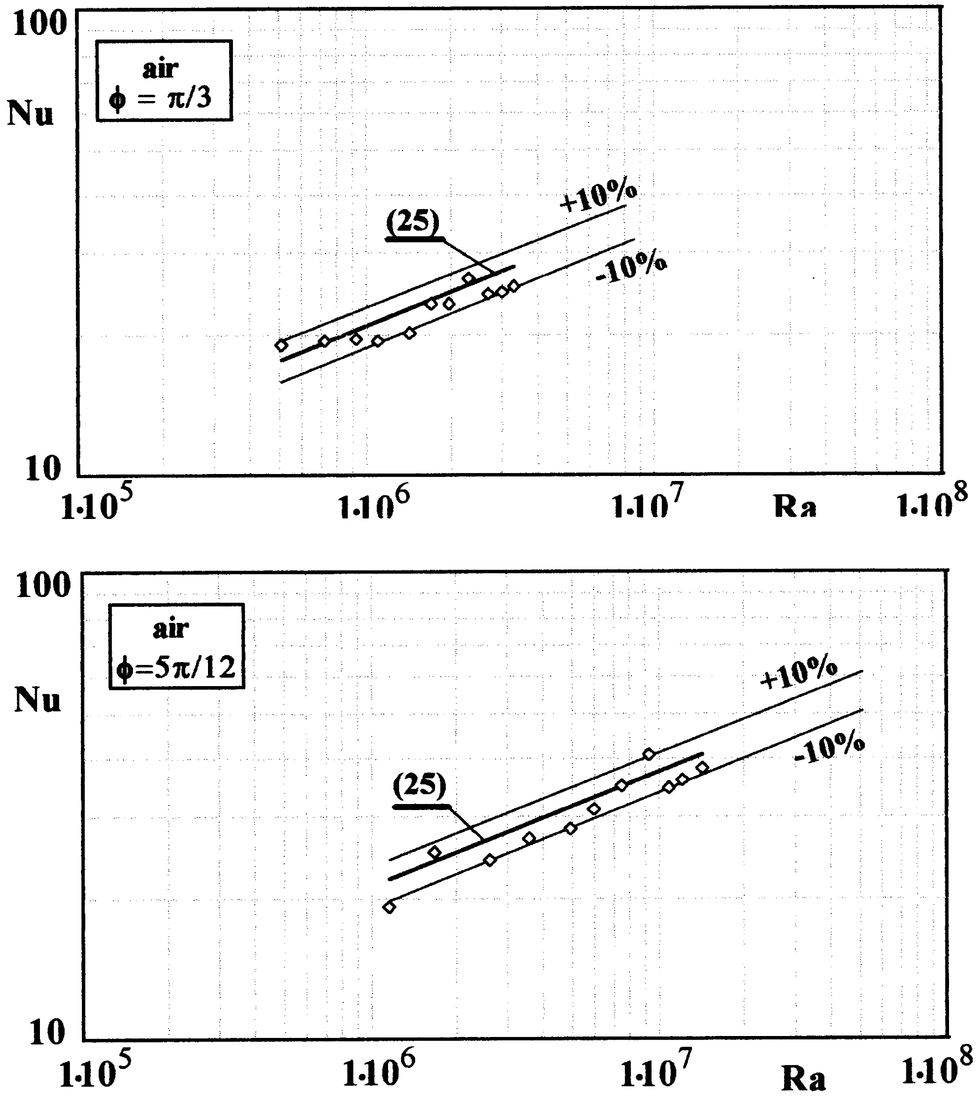


Fig. 7. Experimental results of convective heat transfer from conical surfaces obtained in the second set-up in air.

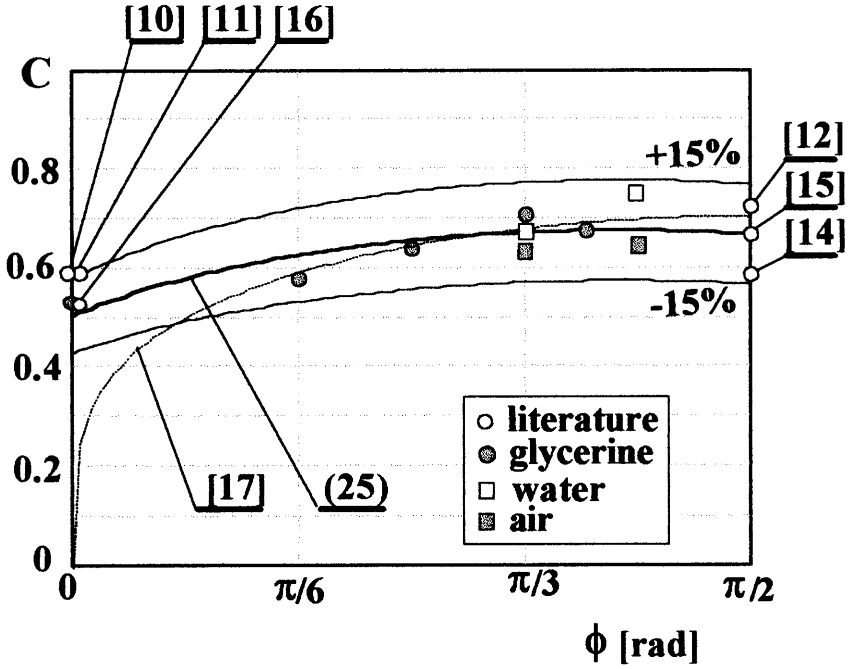


Fig. 8. Comparison of experimental results with theoretical solution.

$$C = \frac{0.710 \cdot \cos^{1/4}(\pi/2-\phi)}{[1+(0.352/Pr)^{9/16}]^{4/9}}. \quad (45)$$

Besides our own results (35)–(43), for extreme cases; circular horizontal plate ($\pi = 0$) and vertical cylinder ($\phi = \pi/2$), selected results of other authors are also presented in Fig. 8.

$$Nu_D = 0.70 \cdot Ra_D^{1/4} \quad \text{or}$$

$$Nu_R = 0.589 \cdot Ra_R^{1/4}, \quad 2 \cdot 10^5 < Ra_D < 4 \cdot 10^7,$$

$$\text{round plate}, \quad \phi = 0 \quad [10] \quad (46)$$

$$Nu_D = 0.711 \cdot Ra_D^{1/4} \quad \text{or}$$

$$Nu_R = 0.598 \cdot Ra_R^{1/4}, \quad Ra_D < 4 \cdot 10^7,$$

$$\text{water, glycerine, disk}, \quad \phi = 0 \quad [11] \quad (47)$$

$$Nu_H = 0.726 \cdot Ra_H^{1/4},$$

$$2 \cdot 10^8 < Ra_H < 4 \cdot 10^8, \quad (48)$$

$$\text{water, ethylene glycerol, cylinder}, \quad \phi = \pi/2 \quad [12]$$

$$Nu_H = \frac{4}{3}\left[\frac{7 \cdot Gr_H Pr^2}{5 \cdot (20+27 \cdot Pr)}\right]^{1/4} + \frac{4}{35}\left(\frac{273+315 \cdot Pr}{64+63 \cdot Pr}\right)\frac{H}{D},$$

$$\text{solution, cylinder}, \quad \phi = \pi/2 \quad [13] \quad (49)$$

$$Nu_H = 0.59 \cdot Ra_H^{1/4}, \quad 10^4 < Ra_H < 10^9,$$

$$\text{air, liquids, cylinder}, \quad \phi = \pi/2 \quad [14] \quad (50)$$

$$Nu_H = 0.67 \cdot Ra_H^{1/4}, \quad 10^7 < Ra_H < 10^9,$$

$$\text{air, cylinder}, \quad \phi = \pi/2 \quad [15]. \quad (51)$$

The analysis of Fig. 8 indicates, that experimental results differ from the theoretical solution by less than 15%. This, by 7% (the first stand) and 5% (the second stand) method error, can be regarded as a positive result of the experimental verification.

Some results of visualisation experiments performed in glycerine at the first set-up are presented in Fig 9. The visualisation method is described in [8, 11].

Results illustrated in Fig. 9 refer to cones of $\phi = \pi/6$, $\phi = 1/4$, $\phi = \pi/3$ and $\phi = \pi/8$ in the same conditions of convective heat transfer given by the Rayleigh number $Ra = 3.5 \cdot 10^5$.

## 5. Conclusions

Analytical solutions obtained for vertical isothermal cones, in contrary to solutions of other authors [9] are more universal. They enclose, beside horns, also flat isothermal circular surfaces and vertical cylinders. This solution for angle $\pi/6$–$\pi/2$ is in agreement with the equation given by Steward [17].

Experimental verification in glycerine, water and in the air on two investigation stands, for cones of six various

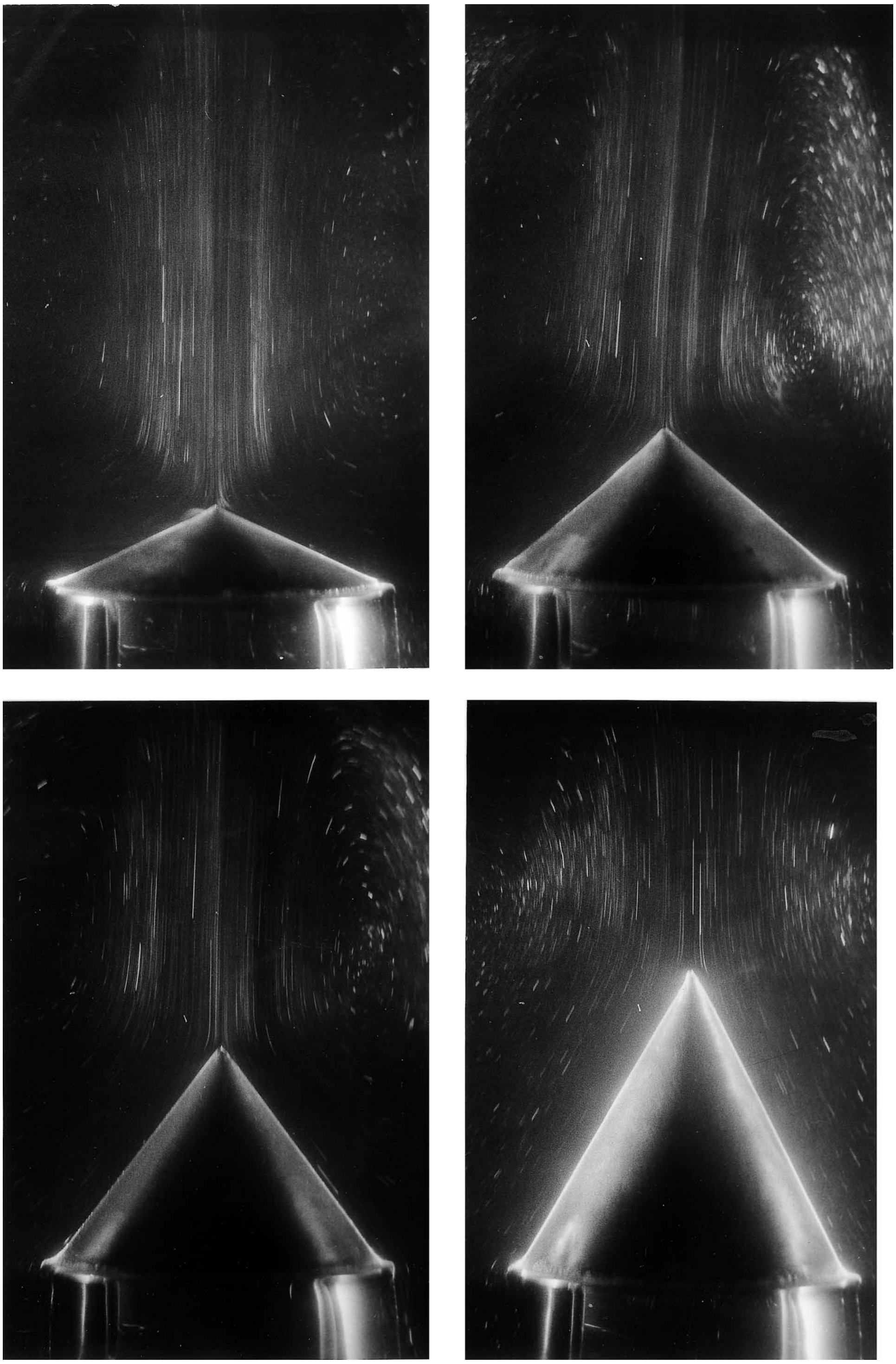

Fig. 9. Photographs of natural convection heat transfer from isothermal cone of the inclination angle: (a) $\phi = \pi/6$; (b) $\phi = \pi/4$; (c) $\phi = \pi/3$; and (d) $\phi = 3 \cdot \pi/8$ performed in glycerine at the first set-up.

inclination angles ($\phi$) and for circular horizontal plates confirmed the correctness of the solution obtained.

## Acknowledgements

This research was partially supported by Scientific Research Grant KBN No. 8T10B 004 13 and Scientific Research Grant of the Chemistry Faculty of the Technical University of Gdańsk under Theses Nos. BW 012233/0951 and BW 012233/098.

## References

[1] P.H. Oosthuizen, E. Donaldson, Free convective heat transfer from vertical cones, Journal of Heat Transfer, Transactions of the ASME, 1972, pp. 330–331.

[2] P.H. Oosthuizen, Free convective heat transfer from horizontal cones, Journal of Heat Transfer, Transactions of the ASME, 1973, pp. 409–410.

[3] Md. Alamgir, Over-all heat transfer from vertical cones in laminar free convection: an approximate method, Journal of Heat Transfer, Transactions of the ASME, 101 (1979) 174–176.

[4] H. Koyama, A. Nakayama, S. Ohsawa and H. Yamada, Theoretical and experimental study of free convection from a vertical frustum of a cone of a finite length, Int. J. Heat Mass Transfer 28(5) (1985) 969–976.

[5] R.G. Hering, Laminar free convection from a non-isothermal cone at low Prandtl numbers, Int. J. Heat Mass Transfer 8 (1965) 1333–1337.

[6] W.M. Lewandowski, P. Kubski, S. Szymański, H. Bieszk, T. Wilczewski, T. Seramak, Quasi steady-state natural convective heat transfer from isothermal circular flat surfaces, International Symposium on Transient Convective Heat Transfer, Begell House, 1997.

[7] W.M. Lewandowski, Natural convection heat transfer from plate of finite dimensions. Int. J. Heat Mass Transfer 34(3) (1991) 875–885.

[8] W.M. Lewandowski, P. Kubski, M.J. Khubeiz, H. Bieszk, T. Wilczewski, S. Szymański, Theoretical and experimental study of natural convection heat transfer from isothermal hemisphere, Int. J. Heat Mass Transfer 40(1) (1997) 101–109.

[9] S.W. Churchill, Free convection around immersed bodies, 2.5 Single-Phase Convective Heat Transfer, Hemisphere, 1983.

[10] M. Al-Arabi, M. El-Riedy, Natural convection heat transfer from isothermal horizontal plates of different shapes, Int. J. Heat Mass Transfer, 19(5) (1976) 1399–1414.

[11] W.M. Lewandowski, H. Bieszk, J. Cieśliński, Free convection from horizontal screened plates, Wärme- und Stoffübertragung, 27 (1992) 481–488.

[12] Y.S. Touloukian, G.A. Hawkins, M. Jakob, Heat transfer by free convection from heated vertical surfaces to liquids, Transactions of the ASME, 70 (1948), 13–18.

[13] E.J. Le Fevre, A.J. Ede, Laminar free convection from the outer surface of a vertical circular cylinder, Appl. Mech., Proc. Int. Congr., 9th Brussels 4 (1957) 175–183.

[14] W.H. McAdams, Heat Transmission, McGraw-Hill, New York, 1954.

[15] E. Griffits, A.H. Davis, The transmission of heat by radiation and convection, Spec. Rep. No. 9, D.S.I.R. Food Invest. Bd., London, 1922.

[16] M. Fishenden, O.A. Saunders, Heat Transfer, Oxford University Press, New York, 1950.

[17] W.E. Stewart, Asymptotic calculation of free convection in laminar three-dimensional systems. Int. J. Heat Mass Transfer 14 (1971) 1013–1031.

[18] W.M. Lewandowski, J.M. Khubeiz, P. Kubski, H. Bieszk, T. Wilczewski, S. Szymański, Natural convection heat transfer from complex surface, Int. J. Heat and Mass Transfer 41(12) (1998) 1858–1868.